Model Uncertainty in Operational Risk Modeling

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* This is a joint work with Professor Vytaras Brazauskas at the University of Wisconsin-Milwaukee

- Example: Bankruptcy of Barings Bank 1995
  \$1.3 billion loss due to Unauthorized Trading
- Operational Risk - Definition
  "The risk of loss arising from inadequate or failed internal processes, people or systems or from external events."
- Basel II/III requires sound management of operational risk.

- Thresholds vary
- Threshold treatment
  - (Cavallo 2012) Truncated vs. Shifted
  - (Shevchenko 2011) Bayesian
  - (Luo 2007) Unbiased vs. Shifted vs. Naive
  - (Chernobai 2006) Truncated vs. Naive

- Model uncertainty
  - (Stricker 2014) Insurance Enterprise Risk
  - (Luo 2007) Operational Risk
  - (Cont 2006) Derivative Instruments
- Robustness
  - (Hansen and Sargent 2007) Macroeconomics

- Naive: d ignored
  - $\mu_{\mathcal{N}}$: location of raw data
  - $\sigma_{\mathcal{N}}$: scale of raw data
- Shifted: $X_i - d$
  - μ_s: location of log shifted data
  - σ_s: scale of log shifted data
- Folded: $\dfrac{X_i}{d}$
  - μ_f: 0
  - σ_f: scale of log folded data
- Truncated: $\dfrac{f(X_i)}{1 - F(d)}$
  - μ_t: location of log data
  - σ_t: scale of log data

Completely ignore the threshold to fit LN

- Data: $X_1, \ldots, X_n$
- Fitted model: $LN(\mu, \sigma)$
- Parameter estimates:

$$\hat{\mu}_{\mathcal{N}} = \frac{1}{n} \sum_{i=1}^{n} \ln(X_i)$$

$$\hat{\sigma}^2_{\mathcal{N}} = \frac{1}{n} \sum_{i=1}^{n} \left[\ln(X_i) - \hat{\mu}_{\mathcal{N}}\right]^2$$

- Data: $X_1 - d, \ldots, X_n - d$
- Fitted model: $LN\,(\mu, \sigma)$
- Parameter estimates

$$\hat{\mu}_s = \frac{1}{n} \sum_{i=1}^{n} \ln\left(X_i - d\right)$$

$$\hat{\sigma}_s^2 = \frac{1}{n} \sum_{i=1}^{n} \left[\ln\left(X_i - d\right) - \hat{\mu}_s\right]^2$$

- Data: $\ln\left(\frac{X_1}{d}\right), \ldots, \ln\left(\frac{X_n}{d}\right)$
- Fitted model: $LFN\left(\mu, \sigma\right)$
- Parameter estimates

$$\hat{\sigma}_f^2 = \frac{1}{n}\sum_{i=1}^{n}\left[\ln\left(\frac{X_i}{d}\right)\right]^2$$

while $\hat{\mu}_f = 0$

Properly take into account the truncation

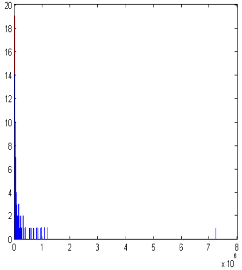
- Data: $X_1|X_1 > d, \ldots, X_n|X_n > d$
- Fitted model: Truncated $LN(\mu, \sigma)$
- Log-likelihood:

$$L(\mu, \sigma|d) = \prod_{i=1}^{n} \frac{f(X_i)}{1 - F(d)}$$

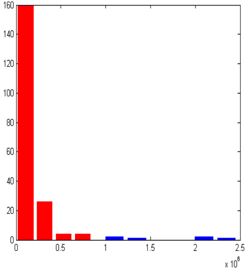
$$\ln L = \sum_{i=1}^{n} \ln f(X_i) - n \ln [1 - F(d)]$$

Maximize the log-likelihood numerically

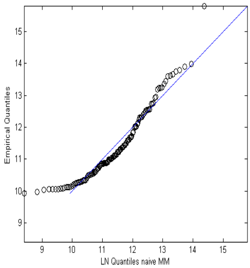
- Simulation study
- Raw data: $LN\left(\mu = 10.755, \sigma = 1.515\right)$
- Sample size: $n = 200$
- Threshold: $d = 20,000$
- Modeling approaches: naive, shifted, folded, truncated
- Monte Carlo: $m = 10,000$
- Parameter estimation: estimate μ, σ, $F\left(d\right)$, and VaR.



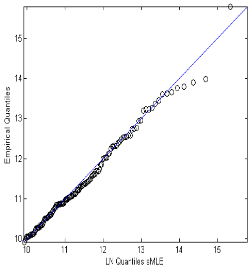
(a) $d = 20,000.$



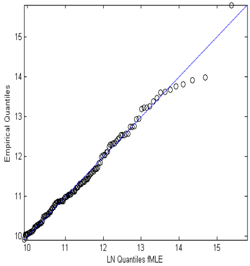
(b) $d = 1,000,000.$



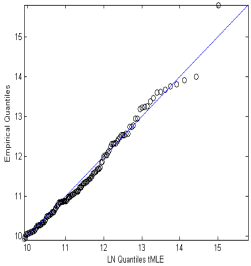
(c) *MM-naive.*



(d) *sMLE.*


Empirical Quantiles
LN Quantiles fMLE
10
11
12
13
14
15

(e) *fMLE.*


Empirical Quantiles
LN Quantiles tMLE
10
11
12
13
14
15

(f) *tMLE.*

- The naive approach

$$\widehat{VaR}_{\mathcal{N}}(q) = e^{\hat{\mu}_{\mathcal{N}} + \hat{\sigma}_{\mathcal{N}} \Phi^{-1}(q)}$$

- The shifted approach

$$\widehat{VaR}_{s}(q) = d + e^{\hat{\mu}_{s} + \hat{\sigma}_{s} \Phi^{-1}(q)}$$

- The folded approach

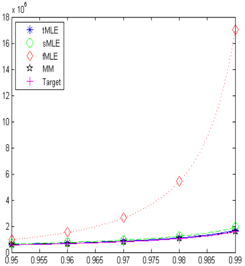
$$\widehat{VaR}_{f}(q) = d * e^{\hat{\sigma}_{f} \Phi^{-1}(q)}$$

- The truncated approach

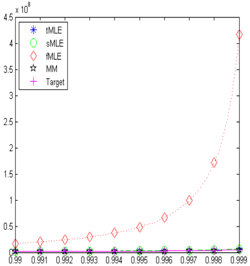
$$\widehat{VaR}_{t}(q) = e^{\hat{\mu}_{t} + \hat{\sigma}_{t} \Phi^{-1}(q)}$$

$10,000$ runs, sample size $n = 200$, $d = 1,000$

Procedure	sMLE	fMLE	tMLE	Target	naive
μ					
median	10.74	0	10.79	10.755	10.82
average	10.75	0	10.80	10.755	10.82
std error	0.10	0	0.10	0	0.09
σ					
median	1.60	4.19	1.51	1.515	1.49
average	1.59	4.19	1.52	1.515	1.49
std error	0.10	0.09	0.10	0	0.08
$F(d)$					
median	0	0.5	0.005	0.0056	0
average	0	0.5	0.006	0.0056	0
std error	0	0	0.0035	0	0
VaR					
95%	6.43	9.8	5.96	5.66	5.84
99%	19	170	16.8	15.9	16.2



(g) $d = 1,000.$

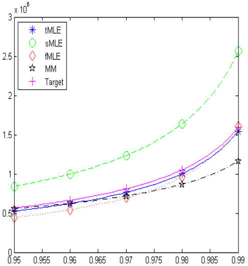


(h) $d = 1,000.$

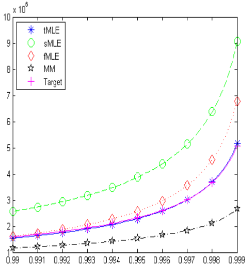
$10,000$ **runs, sample size** $n = 200$, $d = 20,000$

Procedure	sMLE	fMLE	tMLE	Target	naive
μ					
median	10.88	0	10.60	10.755	11.44
average	10.89	0	10.57	10.755	11.45
std error	0.09	0	0.29	0	0.06
σ					
median	1.63	1.88	1.58	1.515	1.09
average	1.66	1.89	1.58	1.515	1.08
std error	0.06	0.06	0.13	0	0.03
$F(d)$					
median	0	0.5	0.33	0.29	0
average	0	0.5	0.33	0.29	0
std error	0	0	0.08	0	0
VaR					
95%	8.41	4.45	5.26	5.66	5.57
99%	25.6	16.1	15.4	15.9	11.7

VaR Assessment Comparisons



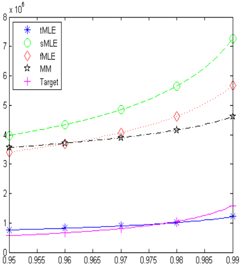
(i) $d = 20,000.$



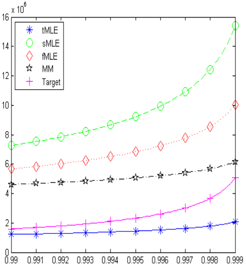
(j) $d = 20,000.$

$10,000$ **runs, sample size** $n = 200$, $d = 1,000,000$

Procedure	sMLE	fMLE	tMLE	Target	naive
μ					
median	12.85	0	14.06	10.755	14.27
average	13.11	0	12.41	10.755	14.47
std error	1.19	0	4.58	0	0.49
σ					
median	0.97	0.57	0.47	1.515	0.34
average	1.09	0.75	0.69	1.515	0.38
std error	0.69	0.49	0.70	0	0.27
$F(d)$					
median	0	0.5	0.23	0.98	0
average	0	0.5	0.37	0.98	0
std error	0	0	0	0	0
VaR					
95%	39.7	34.1	7.62	5.66	35.7
99%	72.6	56.7	12.2	15.9	46.1



(k) $d = 1,000,000.$



(l) $d = 1,000,000.$

- Naive: overestimate μ, underestimate σ, $F(d)$ and VaR
- sMLE: parameters estimates not comparable, overestimate VaR
- fMLE: parameters estimates not comparable, overestimate VaR
- tMLE: tends to underestimate VaR but close

- Model uncertainty in treatment of threshold and impact on VaR
- Naive: liberal
- sMLE: conservative (good for small d)
- fMLE: conservative (good for medium d)
- tMLE: liberal (good for small and medium d)
- Future work